## Advanced Financial Management

## Tuesday 2 December 2014



## Time allowed

Reading and planning: 15 minutes
Writing:
3 hours
This paper is divided into two sections:
Section A - This ONE question is compulsory and MUST be attempted
Section B - TWO questions ONLY to be attempted
Formulae and tables are on pages 8-12.
Do NOT open this paper until instructed by the supervisor.
During reading and planning time only the question paper may
be annotated. You must NOT write in your answer booklet until
instructed by the supervisor.
This question paper must not be removed from the examination hall.

## Section A - This ONE question is compulsory and MUST be attempted

## 1 Nahara Co and Fugae Co

Nahara Co is a private holding company owned by the government of a wealthy oil-rich country to invest its sovereign funds. Nahara Co has followed a strategy of risk diversification for a number of years by acquiring companies from around the world in many different sectors.

One of Nahara Co's acquisition strategies is to identify and purchase undervalued companies in the airline industry in Europe. A recent acquisition was Fugae Co, a company based in a country which is part of the European Union (EU). Fugae Co repairs and maintains aircraft engines.

A few weeks ago, Nahara Co stated its intention to pursue the acquisition of an airline company based in the same country as Fugae Co. The EU, concerned about this, asked Nahara Co to sell Fugae Co before pursuing any further acquisitions in the airline industry.

## Avem Co's acquisition interest in Fugae Co

Avem Co, a UK-based company specialising in producing and servicing business jets, has approached Nahara Co with a proposal to acquire Fugae Co for $\$ 1,200$ million. Nahara Co expects to receive a premium of at least $30 \%$ on the estimated equity value of Fugae Co , if it is sold.

Given below are extracts from the most recent statements of financial position of both Avem Co and Fugae Co.

|  | Avem Co <br> \$ million | Fugae Co |
| :--- | :---: | :---: |
| Share capital (50c/share) | 800 | 100 |
| Reserves | 3,550 | 160 |
| Non-current liabilities | 2,200 | 380 |
| Current liabilities | 130 | 30 |
| Total capital and liabilities | $\underline{6,680}$ | $\underline{670}$ |

Each Avem Co share is currently trading at $\$ 7 \cdot 50$, which is a multiple of $7 \cdot 2$ of its free cash flow to equity. Avem Co expects that the total free cash flows to equity of the combined company will increase by $\$ 40$ million due to synergy benefits. After adding the synergy benefits of $\$ 40$ million, Avem Co then expects the multiple of the total free cash flow of the combined company to increase to $7 \cdot 5$.

Fugae Co's free cash flow to equity is currently estimated at $\$ 76.5$ million and it is expected to generate a return on equity of $11 \%$. Over the past few years, Fugae Co has returned $77 \cdot 3 \%$ of its annual free cash flow to equity back to Nahara Co, while retaining the balance for new investments.

Fugae Co's non-current liabilities consist entirely of $\$ 100$ nominal value bonds which are redeemable in four years at the nominal value, on which the company pays a coupon of $5 \cdot 4 \%$. The debt is rated at $\mathrm{B}+$ and the credit spread on $B+$ rated debt is 80 basis points above the risk-free rate of return.

## Proposed luxury transport investment project by Fugae Co

In recent years, the country in which Fugae Co is based has been expanding its tourism industry and hopes that this industry will grow significantly in the near future. At present tourists normally travel using public transport and taxis, but there is a growing market for luxury travel. If the tourist industry does expand, then the demand for luxury travel is expected to grow rapidly. Fugae Co is considering entering this market through a four-year project. The project will cease after four years because of increasing competition.

The initial cost of the project is expected to be $\$ 42,000,000$ and it is expected to generate the following after-tax cash flows over its four-year life:

| Year | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Cash flows (\$000s) | $3,277.6$ | $16,134.3$ | $36,504.7$ | $35,683.6$ |

The above figures are based on the tourism industry expanding as expected. However, it is estimated that there is a $25 \%$ probability that the tourism industry will not grow as expected in the first year. If this happens, then the present value of the project's cash flows will be $50 \%$ of the original estimates over its four-year life.

It is also estimated that if the tourism industry grows as expected in the first year, there is still a $20 \%$ probability that the expected growth will slow down in the second and subsequent years, and the present value of the project's cash flows would then be $40 \%$ of the original estimates in each of these years.

Lumi Co, a leisure travel company, has offered $\$ 50$ million to buy the project from Fugae $C o$ at the start of the second year. Fugae Co is considering whether having this choice would add to the value of the project.

If Fugae Co is bought by Avem Co after the project has begun, it is thought that the project will not result in any additional synergy benefits and will not generate any additional value for the combined company, above any value the project has already generated for Fugae Co.

Although there is no beta for companies offering luxury forms of travel in the tourist industry, Reka Co, a listed company, offers passenger transportation services on coaches, trains and luxury vehicles. About $15 \%$ of its business is in the luxury transport market and Reka Co's equity beta is $1 \cdot 6$. It is estimated that the asset beta of the non-luxury transport industry is 0.80 . Reka Co's shares are currently trading at $\$ 4.50$ per share and its debt is currently trading at $\$ 105$ per $\$ 100$. It has 80 million shares in issue and the book value of its debt is $\$ 340$ million. The debt beta is estimated to be zero.

General information
The corporation tax rate applicable to all companies is $20 \%$. The risk-free rate is estimated to be $4 \%$ and the market risk premium is estimated to be $6 \%$.

## Required:

(a) Discuss whether or not Nahara Co's acquisition strategies, of pursuing risk diversification and of purchasing undervalued companies, can be valid.
(7 marks)
(b) Discuss why the European Union (EU) may be concerned about Nahara Co's stated intention and how selling Fugae Co could reduce this concern.
(c) Prepare a report for the Board of Directors of Avem Co, which:
(i) Estimates the additional value created for Avem Co, if it acquires Fugae Co without considering the luxury transport project;
(10 marks)
(ii) Estimates the additional value of the luxury transport project to Fugae Co, both with and without the offer from Lumi Co;
(18 marks)
(iii) Evaluates the benefit attributable to Avem Co and Fugae Co from combining the two companies with and without the project, and concludes whether or not the acquisition is beneficial. The evaluation should include any assumptions made.
(7 marks)
Professional marks will be awarded in part (c) for the format, structure and presentation of the report.

## Section B - TWO questions ONLY to be attempted

2 Keshi Co is a large multinational company with a number of international subsidiary companies. A centralised treasury department manages Keshi Co and its subsidiaries' borrowing requirements, cash surplus investment and financial risk management. Financial risk is normally managed using conventional derivative products such as forwards, futures, options and swaps.

Assume it is 1 December 2014 today and Keshi Co is expecting to borrow \$18,000,000 on 1 February 2015 for a period of seven months. It can either borrow the funds at a variable rate of LIBOR plus 40 basis points or a fixed rate of $5 \cdot 5 \%$. LIBOR is currently $3.8 \%$ but Keshi Co feels that this could increase or decrease by $0.5 \%$ over the coming months due to increasing uncertainty in the markets.

The treasury department is considering whether or not to hedge the $\$ 18,000,000$, using either exchange-traded March options or over-the-counter swaps offered by Rozu Bank.

The following information and quotes for $\$$ March options are provided from an appropriate exchange. The options are based on three-month \$ futures, \$1,000,000 contract size and option premiums are in annual \%.

| March calls | Strike price | March puts |
| :---: | :---: | :---: |
| 0.882 | 95.50 | 0.662 |
| 0.648 | 96.00 | 0.902 |

Option prices are quoted in basis points at 100 minus the annual \% yield and settlement of the options contracts is at the end of March 2015. The current basis on the March futures price is 44 points; and it is expected to be 33 points on 1 January 2015, 22 points on 1 February 2015 and 11 points on 1 March 2015.

Rozu Bank has offered Keshi Co a swap on a counterparty variable rate of LIBOR plus 30 basis points or a fixed rate of $4.6 \%$, where Keshi Co receives $70 \%$ of any benefits accruing from undertaking the swap, prior to any bank charges. Rozu Bank will charge Keshi Co 10 basis points for the swap.

Keshi Co's chief executive officer believes that a centralised treasury department is necessary in order to increase shareholder value, but Keshi Co's new chief financial officer (CFO) thinks that having decentralised treasury departments operating across the subsidiary companies could be more beneficial. The CFO thinks that this is particularly relevant to the situation which Suisen Co, a company owned by Keshi Co, is facing.

Suisen Co operates in a country where most companies conduct business activities based on Islamic finance principles. It produces confectionery products including chocolates. It wants to use Salam contracts instead of commodity futures contracts to hedge its exposure to price fluctuations of cocoa. Salam contracts involve a commodity which is sold based on currently agreed prices, quantity and quality. Full payment is received by the seller immediately, for an agreed delivery to be made in the future.

## Required:

(a) Based on the two hedging choices Keshi Co is considering, recommend a hedging strategy for the $\$ 18,000,000$ borrowing. Support your answer with appropriate calculations and discussion. (15 marks)
(b) Discuss how a centralised treasury department may increase value for Keshi Co and the possible reasons for decentralising the treasury department.
(6 marks)
(c) Discuss the key differences between a Salam contract, under Islamic finance principles, and futures contracts.

3 Riviere Co is a small company based in the European Union (EU). It produces high quality frozen food which it exports to a small number of supermarket chains located within the EU as well. The EU is a free trade area for trade between its member countries.

Riviere Co finds it difficult to obtain bank finance and relies on a long-term strategy of using internally generated funds for new investment projects. This constraint means that it cannot accept every profitable project and often has to choose between them.

Riviere $C o$ is currently considering investment in one of two mutually exclusive food production projects: Privi and Drugi. Privi will produce and sell a new range of frozen desserts exclusively within the EU. Drugi will produce and sell a new range of frozen desserts and savoury foods to supermarket chains based in countries outside the EU. Each project will last for five years and the following financial information refers to both projects.

Project Drugi, annual after-tax cash flows expected at the end of each year (€000s)

|  | Current | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Year | (11,840) | 1,230 | 1,680 | 4,350 | 10,240 |$\quad 2,200$

Both projects' net present value has been calculated based on Riviere Co's nominal cost of capital of $10 \%$. It can be assumed that both projects' cash flow returns are normally distributed and the annual standard deviation of project Drugi's present value of after-tax cash flows is estimated to be $€ 400,000$. It can also be assumed that all sales are made in $€$ (Euro) and therefore the company is not exposed to any foreign exchange exposure.

Notwithstanding how profitable project Drugi may appear to be, Riviere Co's board of directors is concerned about the possible legal risks if it invests in the project because they have never dealt with companies outside the EU before.

## Required:

(a) Discuss the aims of a free trade area, such as the European Union (EU), and the possible benefits to Riviere Co of operating within the EU.
(5 marks)
(b) Calculate the figures which have not been provided for project Drugi and recommend which project should be accepted. Provide a justification for the recommendation and explain what the value at risk measures.
(13 marks)
(c) Discuss the possible legal risks of investing in project Drugi which Riviere Co may be concerned about and how these may be mitigated.

4 Kamala Co, a listed company, manufactures parts and machinery for the construction industry. About five years ago, Kamala Co started to manufacture parts and machinery for hospitals and companies engaged in biomedical research using largely the same manufacturing and processing systems it already had in place. In 2011, a young and ambitious chief executive officer (CEO) took over the running of the company.

With the publication of the latest financial statements for the year to 30 November 2014, the CEO made a brief statement and it includes the following two points:

- The CEO was very pleased with growth in the financial ratios provided and sales revenue from 2012 to 2014. More pleasing was growth in the share price, which increased even faster than the growth in the market index, suggesting that Kamala Co has been a successful company.
- The CEO expressed a desire to make Kamala Co the leading manufacturer of parts and machinery for the construction industry by acquiring a major rival manufacturer in 2015, and financing the acquisition through an issue of a new bond and a small rights issue.

An analyst, after examining the recent financial statements and the two points above, was less positive about Kamala Co's future prospects.

Given below are extracts from the recent financial statements, some ratios, and other financial information for Kamala Co.

| Kamala Co <br> Year ending 30 November (all amounts in \$m) |  |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Sales revenue | 3,760 | 4,054 | 5,230 |
| Operating profit | 714 | 819 | 1,098 |
| Finance costs | 97 | 168 | 269 |
| Profit before tax | 617 | 651 | 829 |
| Taxation | 154 | 163 | 207 |
| Profit for the year | 463 | 488 | 622 |
| Dividends | 139 | 137 | 152 |


| Kamala Co |  |  |  |
| :---: | :---: | :---: | :---: |
| Year ending 30 November (all amounts in \$m) |  |  |  |
|  | 2012 | 2013 | 2014 |
| Total non-current assets | 3,962 | 5,507 | 7,669 |
| Total current assets | 980 | 1,410 | 1,880 |
| Total non-current and current assets | 4,942 | 6,917 | 9,549 |
| Equity |  |  |  |
| Ordinary shares (\$0.25) | 750 | 750 | 750 |
| Reserves | 1,476 | 1,827 | 2,297 |
| Total equity | 2,226 | 2,577 | 3,047 |
| Non-current liabilities |  |  |  |
| Bank loans | 476 | 1,176 | 1,316 |
| Bonds | 1,008 | 1,008 | 2,218 |
| Total non-current liabilities | 1,484 | 2,184 | 3,534 |
| Current liabilities |  |  |  |
| Trade and other payables | 1,232 | 1,540 | 2,016 |
| Bank overdraft | - | 616 | 952 |
| Total current liabilities | 1,232 | 2,156 | 2,968 |
| Total non-current and current liabilities | 2,716 | 4,340 | 6,502 |

Kamala Co: By activity
Year ending 30 November (all amounts in \$m)

|  | 2012 | 2013 | 2014 |
| :--- | ---: | ---: | ---: |
| Sales revenue <br> $\quad$ Construction |  |  |  |
| $\quad$ Hospitals and biomedical | 1,420 | 2,644 | 3,660 |
| Operating profit |  | 1,410 | 1,570 |
| $\quad$ Construction |  |  |  |
| $\quad$ Hospitals and biomedical | 254 | 489 | 693 |
| Ratios: Kamala Co | 330 | 405 |  |
|  |  |  |  |
|  | 2012 | 2013 | 2014 |
| Operating profit margin | $19 \cdot 0 \%$ | $20 \cdot 2 \%$ | $21 \cdot 0 \%$ |
| Dividend cover | $3 \cdot 3$ | $3 \cdot 6$ | $4 \cdot 1$ |
| Earnings per share | $15 \cdot 4 \mathrm{c}$ | $16 \cdot 3 \mathrm{c}$ | $20 \cdot 7 \mathrm{c}$ |
| Gearing [(debt/debt + equity)] | $40 \%$ | $46 \%$ | $54 \%$ |
|  |  |  |  |

Other financial information

|  | 30 | November | 30 November |
| :--- | :---: | :---: | :---: | 30 November

Kamala Co's cost of capital is estimated to be $10 \%$. The company's corporation tax rate is $25 \%$.

## Required:

(a) Discuss the advantages and drawbacks of using the economic value added (EVA ${ }^{T M}$ ) technique to assess a company's performance.
(6 marks)
(b) Estimate Kamala Co's EVA ${ }^{\text {TM }}$ for the years ending 30 November 2013 and 30 November 2014. (5 marks)
(c) Evaluate Kamala Co's performance and conclude whether the analyst's opinion or the chief executive officer's opinion has the greater validity. Include any additional ratio and activity trends, and share price analysis, which are deemed to be relevant to the evaluation.

## Formulae

Modigliani and Miller Proposition 2 (with tax)

$$
k_{e}=k_{e}^{i}+(1-T)\left(k_{e}^{i}-k_{d}\right) \frac{V_{d}}{V_{e}}
$$

The Capital Asset Pricing Model

$$
\mathrm{E}\left(\mathrm{r}_{\mathrm{i}}\right)=\mathrm{R}_{\mathrm{f}}+\beta_{\mathrm{i}}\left(\mathrm{E}\left(\mathrm{r}_{\mathrm{m}}\right)-\mathrm{R}_{\mathrm{f}}\right)
$$

The asset beta formula

$$
\beta_{\mathrm{a}}=\left[\frac{\mathrm{V}_{\mathrm{e}}}{\left(\mathrm{~V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{e}}\right]+\left[\frac{\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})}{\left(\mathrm{V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{d}}\right]
$$

The Growth Model

$$
P_{o}=\frac{D_{0}(1+g)}{\left(r_{e}-g\right)}
$$

Gordon's growth approximation

$$
\mathrm{g}=\mathrm{br} r_{\mathrm{e}}
$$

The weighted average cost of capital

$$
\text { WACC }=\left[\frac{V_{e}}{V_{e}+V_{d}}\right] k_{e}+\left[\frac{V_{d}}{V_{e}+V_{d}}\right] k_{d}(1-T)
$$

The Fisher formula

$$
(1+i)=(1+r)(1+h)
$$

Purchasing power parity and interest rate parity

$$
S_{1}=S_{0} \times \frac{\left(1+h_{c}\right)}{\left(1+h_{b}\right)} \quad F_{0}=S_{0} \times \frac{\left(1+\mathrm{i}_{\mathrm{c}}\right)}{\left(1+\mathrm{i}_{\mathrm{b}}\right)}
$$

Modified Internal Rate of Return

$$
\operatorname{MIRR}=\left[\frac{P V_{R}}{P V_{I}}\right]^{\frac{1}{n}}\left(1+r_{e}\right)-1
$$

The Black-Scholes option pricing model

$$
\mathrm{c}=\mathrm{P}_{\mathrm{a}} \mathrm{~N}\left(\mathrm{~d}_{1}\right)-\mathrm{P}_{\mathrm{e}} \mathrm{~N}\left(\mathrm{~d}_{2}\right) \mathrm{e}^{-r t}
$$

Where:

$$
\begin{aligned}
& d_{1}=\frac{\ln \left(P_{a} / P_{e}\right)+\left(r+0.5 s^{2}\right) t}{s \sqrt{t}} \\
& d_{2}=d_{1}-s \sqrt{t}
\end{aligned}
$$

The Put Call Parity relationship

$$
p=c-P_{a}+P_{e} e^{-r t}
$$

## Present Value Table

Present value of 1 i.e. $(1+r)^{-n}$
$\begin{array}{ll}\text { Where } & r=\text { discount rate } \\ n & =\text { number of periods until payment }\end{array}$

Discount rate (r)
Periods

| (n) | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ | $8 \%$ | $9 \%$ | $10 \%$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 0.980 | 0.961 | 0.943 | 0.925 | 0.907 | 0.890 | 0.873 | 0.857 | 0.842 | 0.826 | 2 |
| 3 | 0.971 | 0.942 | 0.915 | 0.889 | 0.864 | 0.840 | 0.816 | 0.794 | 0.772 | 0.751 | 3 |
| 4 | 0.961 | 0.924 | 0.888 | 0.855 | 0.823 | 0.792 | 0.763 | 0.735 | 0.708 | 0.683 | 4 |
| 5 | 0.951 | 0.906 | 0.863 | 0.822 | 0.784 | 0.747 | 0.713 | 0.681 | 0.650 | 0.621 | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0.942 | 0.888 | 0.837 | 0.790 | 0.746 | 0.705 | 0.666 | 0.630 | 0.596 | 0.564 | 6 |
| 7 | 0.933 | 0.871 | 0.813 | 0.760 | 0.711 | 0.665 | 0.623 | 0.583 | 0.547 | 0.513 | 7 |
| 8 | 0.923 | 0.853 | 0.789 | 0.731 | 0.677 | 0.627 | 0.582 | 0.540 | 0.502 | 0.467 | 8 |
| 9 | 0.914 | 0.837 | 0.766 | 0.703 | 0.645 | 0.592 | 0.544 | 0.500 | 0.460 | 0.424 | 9 |
| 10 | 0.905 | 0.820 | 0.744 | 0.676 | 0.614 | 0.558 | 0.508 | 0.463 | 0.422 | 0.386 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 0.896 | 0.804 | 0.722 | 0.650 | 0.585 | 0.527 | 0.475 | 0.429 | 0.388 | 0.350 | 11 |
| 12 | 0.887 | 0.788 | 0.701 | 0.625 | 0.557 | 0.497 | 0.444 | 0.397 | 0.356 | 0.319 | 12 |
| 13 | 0.879 | 0.773 | 0.681 | 0.601 | 0.530 | 0.469 | 0.415 | 0.368 | 0.326 | 0.290 | 13 |
| 14 | 0.870 | 0.758 | 0.661 | 0.577 | 0.505 | 0.442 | 0.388 | 0.340 | 0.299 | 0.263 | 14 |
| 15 | 0.861 | 0.743 | 0.642 | 0.555 | 0.481 | 0.417 | 0.362 | 0.315 | 0.275 | 0.239 | 15 |


| (n) | $11 \%$ | $12 \%$ | $13 \%$ | $14 \%$ | $15 \%$ | $16 \%$ | $17 \%$ | $18 \%$ | $19 \%$ | $20 \%$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | 0.862 | 0.855 | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 0.812 | 0.797 | 0.783 | 0.769 | 0.756 | 0.743 | 0.731 | 0.718 | 0.706 | 0.694 | 2 |
| 3 | 0.731 | 0.712 | 0.693 | 0.675 | 0.658 | 0.641 | 0.624 | 0.609 | 0.593 | 0.579 | 3 |
| 4 | 0.659 | 0.636 | 0.613 | 0.592 | 0.572 | 0.552 | 0.534 | 0.516 | 0.499 | 0.482 | 4 |
| 5 | 0.593 | 0.567 | 0.543 | 0.519 | 0.497 | 0.476 | 0.456 | 0.437 | 0.419 | 0.402 | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0.535 | 0.507 | 0.480 | 0.456 | 0.432 | 0.410 | 0.390 | 0.370 | 0.352 | 0.335 | 6 |
| 7 | 0.482 | 0.452 | 0.425 | 0.400 | 0.376 | 0.354 | 0.333 | 0.314 | 0.296 | 0.279 | 7 |
| 8 | 0.434 | 0.404 | 0.376 | 0.351 | 0.327 | 0.305 | 0.285 | 0.266 | 0.249 | 0.233 | 8 |
| 9 | 0.391 | 0.361 | 0.333 | 0.308 | 0.284 | 0.263 | 0.243 | 0.225 | 0.209 | 0.194 | 9 |
| 10 | 0.352 | 0.322 | 0.295 | 0.270 | 0.247 | 0.227 | 0.208 | 0.191 | 0.176 | 0.162 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 0.317 | 0.287 | 0.261 | 0.237 | 0.215 | 0.195 | 0.178 | 0.162 | 0.148 | 0.135 | 11 |
| 12 | 0.286 | 0.257 | 0.231 | 0.208 | 0.187 | 0.168 | 0.152 | 0.137 | 0.124 | 0.112 | 12 |
| 13 | 0.258 | 0.229 | 0.204 | 0.182 | 0.163 | 0.145 | 0.130 | 0.116 | 0.104 | 0.093 | 13 |
| 14 | 0.232 | 0.205 | 0.181 | 0.160 | 0.141 | 0.125 | 0.111 | 0.099 | 0.088 | 0.078 | 14 |
| 15 | 0.209 | 0.183 | 0.160 | 0.140 | 0.123 | 0.108 | 0.095 | 0.084 | 0.074 | 0.065 | 15 |

## Annuity Table

Present value of an annuity of 1 i.e. $\frac{1-(1+r)^{-n}}{r}$

$$
\begin{array}{ll}
\text { Where } & r=\text { discount rate } \\
& n=\text { number of periods }
\end{array}
$$

## Discount rate (r)

Periods

| ( n ) | 1\% | 2\% | 3\% | 4\% | 5\% | 6\% | 7\% | 8\% | 9\% | 10\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 1.970 | 1.942 | 1.913 | 1.886 | 1.859 | 1.833 | 1.808 | 1.783 | 1.759 | 1.736 | 2 |
| 3 | 2.941 | $2 \cdot 884$ | $2 \cdot 829$ | $2 \cdot 775$ | $2 \cdot 723$ | $2 \cdot 673$ | $2 \cdot 624$ | $2 \cdot 577$ | $2 \cdot 531$ | $2 \cdot 487$ | 3 |
| 4 | 3.902 | 3.808 | $3 \cdot 717$ | 3.630 | 3.546 | $3 \cdot 465$ | $3 \cdot 387$ | $3 \cdot 312$ | 3.240 | $3 \cdot 170$ | 4 |
| 5 | $4 \cdot 853$ | $4 \cdot 713$ | 4.580 | $4 \cdot 452$ | $4 \cdot 329$ | $4 \cdot 212$ | 4.100 | 3.993 | 3.890 | 3.791 | 5 |
| 6 | $5 \cdot 795$ | $5 \cdot 601$ | $5 \cdot 417$ | $5 \cdot 242$ | 5.076 | 4.917 | $4 \cdot 767$ | $4 \cdot 623$ | $4 \cdot 486$ | $4 \cdot 355$ | 6 |
| 7 | $6 \cdot 728$ | $6 \cdot 472$ | 6.230 | 6.002 | $5 \cdot 786$ | $5 \cdot 582$ | $5 \cdot 389$ | $5 \cdot 206$ | 5.033 | $4 \cdot 868$ | 7 |
| 8 | 7.652 | 7.325 | 7.020 | 6.733 | 6.463 | 6.210 | 5.971 | $5 \cdot 747$ | $5 \cdot 535$ | $5 \cdot 335$ | 8 |
| 9 | 8.566 | 8.162 | 7.786 | 7.435 | $7 \cdot 108$ | $6 \cdot 802$ | $6 \cdot 515$ | $6 \cdot 247$ | 5.995 | $5 \cdot 759$ | 9 |
| 10 | $9 \cdot 471$ | 8.983 | 8.530 | $8 \cdot 111$ | $7 \cdot 722$ | $7 \cdot 360$ | $7 \cdot 024$ | $6 \cdot 710$ | 6.418 | $6 \cdot 145$ | 10 |
| 11 | $10 \cdot 368$ | 9.787 | $9 \cdot 253$ | $8 \cdot 760$ | $8 \cdot 306$ | 7.887 | 7.499 | $7 \cdot 139$ | 6.805 | 6.495 | 11 |
| 12 | $11 \cdot 255$ | $10 \cdot 575$ | 9.954 | $9 \cdot 385$ | 8.863 | 8.384 | 7.943 | 7.536 | $7 \cdot 161$ | 6.814 | 12 |
| 13 | $12 \cdot 134$ | $11 \cdot 348$ | $10 \cdot 635$ | 9.986 | $9 \cdot 394$ | 8.853 | 8.358 | 7.904 | 7.487 | $7 \cdot 103$ | 13 |
| 14 | 13.004 | $12 \cdot 106$ | 11.296 | $10 \cdot 563$ | 9.899 | 9.295 | $8 \cdot 745$ | 8.244 | 7.786 | 7.367 | 14 |
| 15 | 13.865 | $12 \cdot 849$ | 11.938 | $11 \cdot 118$ | $10 \cdot 380$ | $9 \cdot 712$ | $9 \cdot 108$ | 8.559 | 8.061 | $7 \cdot 606$ | 15 |
| ( n ) | 11\% | 12\% | 13\% | 14\% | 15\% | 16\% | 17\% | 18\% | 19\% | 20\% |  |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | 0.862 | 0.855 | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 1.713 | 1.690 | 1.668 | 1.647 | 1.626 | 1.605 | 1.585 | 1.566 | 1.547 | 1.528 | 2 |
| 3 | $2 \cdot 444$ | $2 \cdot 402$ | $2 \cdot 361$ | $2 \cdot 322$ | $2 \cdot 283$ | $2 \cdot 246$ | $2 \cdot 210$ | $2 \cdot 174$ | $2 \cdot 140$ | $2 \cdot 106$ | 3 |
| 4 | $3 \cdot 102$ | 3.037 | 2.974 | $2 \cdot 914$ | $2 \cdot 855$ | $2 \cdot 798$ | $2 \cdot 743$ | $2 \cdot 690$ | 2.639 | $2 \cdot 589$ | 4 |
| 5 | 3.696 | 3.605 | 3.517 | 3.433 | 3.352 | 3.274 | $3 \cdot 199$ | $3 \cdot 127$ | 3.058 | 2.991 | 5 |
| 6 | 4.231 | $4 \cdot 111$ | 3.998 | 3.889 | $3 \cdot 784$ | 3.685 | 3.589 | 3.498 | 3.410 | $3 \cdot 326$ | 6 |
| 7 | $4 \cdot 712$ | 4.564 | $4 \cdot 423$ | $4 \cdot 288$ | $4 \cdot 160$ | 4.039 | 3.922 | 3.812 | 3.706 | 3.605 | 7 |
| 8 | $5 \cdot 146$ | 4.968 | 4.799 | $4 \cdot 639$ | 4.487 | $4 \cdot 344$ | $4 \cdot 207$ | 4.078 | 3.954 | 3.837 | 8 |
| 9 | $5 \cdot 537$ | $5 \cdot 328$ | $5 \cdot 132$ | 4.946 | 4.772 | $4 \cdot 607$ | $4 \cdot 451$ | 4.303 | 4.163 | 4.031 | 9 |
| 10 | $5 \cdot 889$ | $5 \cdot 650$ | $5 \cdot 426$ | $5 \cdot 216$ | 5.019 | $4 \cdot 833$ | $4 \cdot 659$ | $4 \cdot 494$ | $4 \cdot 339$ | $4 \cdot 192$ | 10 |
| 11 | $6 \cdot 207$ | 5.938 | 5.687 | $5 \cdot 453$ | $5 \cdot 234$ | 5.029 | 4.836 | 4.656 | $4 \cdot 486$ | 4.327 | 11 |
| 12 | 6.492 | $6 \cdot 194$ | 5.918 | $5 \cdot 660$ | $5 \cdot 421$ | $5 \cdot 197$ | $4 \cdot 988$ | 4.793 | $4 \cdot 611$ | 4.439 | 12 |
| 13 | $6 \cdot 750$ | $6 \cdot 424$ | $6 \cdot 122$ | $5 \cdot 842$ | 5.583 | $5 \cdot 342$ | $5 \cdot 118$ | 4.910 | $4 \cdot 715$ | 4.533 | 13 |
| 14 | 6.982 | 6.628 | $6 \cdot 302$ | $6 \cdot 002$ | $5 \cdot 724$ | $5 \cdot 468$ | $5 \cdot 229$ | 5.008 | 4.802 | $4 \cdot 611$ | 14 |
| 15 | $7 \cdot 191$ | $6 \cdot 811$ | $6 \cdot 462$ | $6 \cdot 142$ | 5.847 | $5 \cdot 575$ | $5 \cdot 324$ | 5.092 | $4 \cdot 876$ | $4 \cdot 675$ | 15 |

Standard normal distribution table

|  | $0 \cdot 00$ | 0.01 | 0.02 | 0.03 | $0 \cdot 04$ | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| $0 \cdot 1$ | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| $0 \cdot 2$ | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | $0 \cdot 1026$ | $0 \cdot 1064$ | $0 \cdot 1103$ | $0 \cdot 1141$ |
| $0 \cdot 3$ | $0 \cdot 1179$ | $0 \cdot 1217$ | $0 \cdot 1255$ | $0 \cdot 1293$ | 0.1331 | $0 \cdot 1368$ | $0 \cdot 1406$ | 0.1443 | $0 \cdot 1480$ | $0 \cdot 1517$ |
| $0 \cdot 4$ | $0 \cdot 1554$ | $0 \cdot 1591$ | $0 \cdot 1628$ | $0 \cdot 1664$ | $0 \cdot 1700$ | $0 \cdot 1736$ | $0 \cdot 1772$ | $0 \cdot 1808$ | $0 \cdot 1844$ | $0 \cdot 1879$ |
| 0.5 | 0.1915 | $0 \cdot 1950$ | $0 \cdot 1985$ | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | $0 \cdot 3023$ | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| $0 \cdot 9$ | $0 \cdot 3159$ | 0.3186 | $0 \cdot 3212$ | 0.3238 | $0 \cdot 3264$ | $0 \cdot 3289$ | 0.3315 | 0.3340 | $0 \cdot 3365$ | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | $0 \cdot 3461$ | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| $1 \cdot 1$ | 0.3643 | 0.3665 | $0 \cdot 3686$ | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| $1 \cdot 2$ | 0.3849 | 0.3869 | $0 \cdot 3888$ | 0.3907 | $0 \cdot 3925$ | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| $1 \cdot 3$ | 0.4032 | 0.4049 | $0 \cdot 4066$ | 0.4082 | $0 \cdot 4099$ | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| $2 \cdot 1$ | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| $2 \cdot 2$ | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| $2 \cdot 3$ | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| $2 \cdot 4$ | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| $2 \cdot 6$ | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| $2 \cdot 9$ | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

This table can be used to calculate $N(d)$, the cumulative normal distribution functions needed for the Black-Scholes model of option pricing. If $d_{i}>0$, add 0.5 to the relevant number above. If $d_{i}<0$, subtract the relevant number above from 0.5 .

## End of Question Paper

